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Branes, orbifolds and the three dimensional $\mathcal{N} = 2$ SCFT in the large N limit

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Abstract: We study the correspondence between the large N limit of $\mathcal{N} = 2$ three dimensional superconformal field theories and M theory on orbifolds of $AdS_4 \times \mathbf{S^7}$. We identify the brane configuration which gives $\mathbf{C^3}/\mathbf{Z_3}$ as a background for the M theory as a Brane Box Model or a (p, q) web model. By taking the orbifold projection on the known spectrum of Kaluza-Klein harmonics of supergravity, we obtain information about the chiral primary operators of the orbifold singularities.

Keywords: M-Theory, D-branes, Supersymmetry and Duality.

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Contents

1. Introduction

It has been proposed in [1] that the large N limit of super conformal field theories (SCFT) can be described by taking the supergravity limit of the superstring compactified on anti-de Sitter (AdS) space. The correlation functions of SCFT having the AdS boundary as its spacetime can be obtained from the dependence of the supergravity action on the asymptotic behavior of fields at the boundary [2, 3, 4]. This way one can get the scaling dimensions of operators of SCFT from the masses of particles in string/M theory. In particular, $\mathcal{N} = 4$ super $SU(N)$ Yang-Mills theory in 4 dimensions is described by type IIB string theory on $AdS_5 \times \mathbf{S^5}$. The gauge group can be replaced by $SO(N)/Sp(N)$ [5] by taking appropriate orientifold operations [6, 7, 8].

There are $\mathcal{N} = 2, 1, 0$ superconformal models in 4 dimensions which have supergravity description when this is compactificatified on orbifolds of $AdS_5 \times \mathbf{S^5}$ [9, 10]. This proposed duality has been tested by studying the Kaluza-Klein (KK) states of supergravity theory on the orbifolds of $AdS_5 \times \mathbf{S^5}$ and by comparing them with the chiral primary operators of the SCFT on the boundary [11]. As we go one step further, the field theory/ M theory duality gives a supergravity theory on AdS_4 or AdS_7 for some superconformal theories in 3 and 6 dimensions, respectively. The maximally supersymmetric theories in 3 and 6 dimensions have been studied recently [6, 12, 13, 14, 15]. The lower supersymmetric case is realized on the worldvolume of M theory at orbifold singularities [16] (See also [17]). Very recently, along the line of [11], the Kaluza-Klein states of supergravity theory on the orbifolds of $AdS_4 \times \mathbf{S^7}$ were studied and compared with the chiral primary operators of the SCFT on the boundary [18]. Furthermore, the

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analysis for orbifolds of $AdS_7 \times \mathbf{S^4}$ was worked out in [19]. Other important developments of the AdS/SCFT duality can be found in [20, 21, 22, 23, 24, 25, 26, 27, 28, 29].

On the other hand, important results have been obtained by studying the aspects of supersymmetric chiral gauge theories derived from the Brane Boxes Models [30, 31, 32]. By considering a $D4$ brane stretched between two pairs of two NS5 branes, the resulting theory is $\mathcal{N} = 2$ in 3 dimensions. As in [32], we show that a specific Brane Box Model is mapped to a configuration with D2 branes at $\mathbf{C^3}/\mathbf{Z_3}$ singularity giving $\mathcal{N} = 2$ in 3 dimensions. Another interesting class of models consists of web configurations where D3 branes are suspended between (1, 0), (0, 1) and (1, 1) webs which is again shown to be mapped to D2 branes at $\mathbf{C^3}/\mathbf{Z_3}$ singularity. We also discuss possible connection between the Brane Box Model and the web configuration.

As a main result of this paper, we investigate the Kaluza-Klein states of supergravity theory on orbifolds of $AdS_4 \times \mathbf{S^7}$. We obtain the chiral primary operators in the superconformal multiplets by using the correspondence between AdS compactifications and SCFT. As pointed out in [32] for a four dimensional theory, our method will allow us to obtain partial information because we consider only the untwisted modes which are accessible in the supergravity approximation which does not give information about the twisted sectors.

In section 2 we review some known results for $\mathcal{N} = 8$ and $\mathcal{N} = 4$ in three dimensions. In section 3 we obtain our results for $\mathcal{N} = 2$ in three dimensions. We identify the SCFT and we determine its spectrum of chiral primary operators.

2. Review on AdS_4 and $\mathcal{N} = 8, 4$ SCFTs

$\mathcal{N} = 8$ **supersymmetric case:** Let us consider M theory on $AdS_4 \times \mathbf{S^7}$ with a 7 form flux of N quanta on $\mathbf{S^7}$. The radii of AdS_4 and $\mathbf{S^7}$ are given by $2R_{AdS_4} = R_{\mathbf{S^7}} = l_p(32\pi^2 N)^{1/6}$. Eleven dimensional supergravity is appropriate for energies of the order of $1/R_{AdS_4}$ if N becomes very large. The bosonic symmetries are given by $SO(3,2) \times SO(8)$. In [1] it was proposed that the conformal theory on N parallel M2 branes on the boundary of AdS_4 is dual to M theory on the above background. The $SO(3,2)$ part of the symmetry of the supergravity side is the conformal group of the SCFT on the boundary while the $SO(8)$ part corresponds to the R symmetry of the boundary SCFT. From the point of view of type IIA string theory, it is known that this SCFT is the strong coupling limit of the 3 dimensional $\mathcal{N} = 8$ $U(N)$ gauge theory on N coincident D2 branes.

Let us study the correspondence between the Kaluza-Klein excitations of supergravity and the chiral primary fields of the SCFT. The spectrum of the Kaluza-Klein harmonics of eleven dimensional supergravity on $AdS_4 \times \mathbf{S^7}$ was analyzed in [33] some time ago. There exist three families of scalar excitations and two families of pseudoscalar excitations. Three of them contain states with only positive m^2 corresponding to irrelevant operators. One family contains states with negative and zero m^2 with masses given by

$$m^2 = \frac{1}{4}k(k-6)\,, \qquad k = 2, 3, \ldots \tag{2.1}$$

They fall into the kth order symmetric traceless representation of $SO(8)$ with unit multiplicity. The scaling dimensions of the corresponding chiral operators [6] in the SCFT side are

$$\Delta = \frac{k}{2}\,, \qquad k = 2, 3, \ldots \tag{2.2}$$

By regarding this as the strong coupling limit of the 3 dimensional $\mathcal{N} = 8$ SYM theory, some of these operators may be identified with operators of the form $\mathrm{Tr}(X^{i_1}X^{i_2}...X^{i_k})$, where the X^i are the scalar fields in the vector multiplet. For $k = 2, \ldots, 5$ these are relevant operators in SCFT, and for $k = 6$ they are marginal. There is one other family of pseudoscalar excitations which contains states with negative and zero m^2, corresponding to relevant and marginal operators in the SCFT. The masses of this family are given by $m^2 = \frac{1}{4}\left((k-1)(k+1)-8\right)\,, \quad k = 1, 2, \ldots$ The k th state transforms in a representation of $SO(8)$ corresponding to the product of a $\mathbf{35_c}$ with $(k-1)$ $\mathbf{8_v}$'s. The dimensions of the corresponding operators in the SCFT are $\Delta = \frac{k+3}{2}, \quad k = 1, 2, \ldots$ For $k = 1$ we have $\mathbf{35_c}$ pseudoscalar relevant operators of dimension 2. In the UV SYM flowing to this SCFT, we can identify these operators with a product of two fermions times $k-1$ scalars, as in [4, 34].

Next we identify one family of vector bosons that contains massless states. The masses of this family are given by $\widetilde{m}^2 = \frac{1}{4}(k^2-1), \quad k = 1, 2, \ldots$ The dimensions of the corresponding 1-form operators in the SCFT are $\Delta = \frac{k+3}{2}, \quad k = 1, 2, \ldots$ The massless vector at $k = 1$ in (2) corresponds to R symmetry current of dimension 2.

$\mathcal{N} = 4$ supersymmetric case: In [16, 15] supergravity duals of 3 dimensional $\mathcal{N} = 4$ theories with ADE global symmetries were analyzed. They can be realized as worldvolume theories of brane configurations in spacetime. The spacetime compactification can be read from the near horizon geometry of the brane configuration. The brane configuration corresponding to the fixed point is the theory on N M2 branes at a $\mathbf{C^2}/\Gamma$ singularity. The near horizon geometry of this brane configuration is $AdS_4 \times \mathbf{S^3} \times_f \mathbf{D_4}/\Gamma$ where the $\mathbf{S^3}$ is fibered over the Γ quotiented four disk. The theory dual to the fixed point is M theory on $AdS_4 \times \mathbf{S^3} \times_f \mathbf{D_4}/\Gamma$ [15].

Only a subset of the allowed projected states will have the right quantum numbers to be in short $\mathcal{N} = 4$ supersymmetry multiplets. The superconformal primaries from the scalar family are given by in terms of $SU(2)_{ADE} \times SU(2)_L \times SU(2)_R$ representation. None of the surviving states from the pseudoscalar tower have the right quantum numbers to be $\mathcal{N} = 4$ SCFT. The massless vector and the graviton correspond to conserved currents in the superconformal field theory. They couple to the global symmetry current in the adjoint of the R symmetry and to the energy momentum respectively.

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3. An orbifold of $AdS_4 \times \mathbf{S^7}$ and $\mathcal{N} = 2$ SCFT

3.1. Singularities and brane configurations

We now take orbifolds[1] of $AdS_4 \times \mathbf{S^7}$. As usual, we want an orbifold action only on $\mathbf{S^7}$. We want to see the near horizon geometry of the M2 branes. We use orbifold construction which gives $\mathcal{N} = 2$ SCFT in 3 dimensions but the transverse space is not compact. To do this we take a brane configuration with M2 branes at $\mathbf{C^3}/\Gamma$ singularity.[2] By the connection of the M theory configuration involving M2 branes and a type IIA vacuum, we can think of type IIA vacuum as a compactification of M theory on $\mathbf{R^{1,9}} \times \mathbf{S^1}$. Then the D2 branes correspond to M2 branes unwrapped around x^{10}. In 11 dimensions the transverse space to the M2 brane is topologically $\mathbf{R^8}$ and we act with $\mathbf{Z_3}$ to form $\mathbf{R^8}/\mathbf{Z_3} = \mathbf{R^2} \times \mathbf{C^3}/\mathbf{Z_3}$. Here $\mathbf{Z_3}$ acts on $\mathbf{C}^3$ by $\zeta\cdot(z_1, z_2, z_3) = (\omega z_1, \omega z_2, \omega z_3)$ where $\omega = \exp(2\pi i/3)$ and ζ is the generator of $\mathbf{Z}_3$. We first reduce the theory from 11 dimensions to 10 dimensions (type IIA string theory) to obtain D2 branes with transverse space $\mathbf{R} \times \mathbf{C^3}/\mathbf{Z_3}$ and we can easily determine the global symmetry.

If we take x^3 to be compact we obtain D2 branes in $\mathbf{S^1} \times \mathbf{C^3}/\mathbf{Z_3}$. The isometry of $\mathbf{S^1}$ is a global symmetry for the field theory on the world volume of the D2 branes and is identified with the $U(1)_R$ symmetry. We then have the global symmetry breaking $SO(8) \to SU(3) \times U(1)_R$ because $\mathbf{C^3}/\mathbf{Z_3}$ is a Calabi-Yau threefold having $SU(3)$ holonomy group.

Here we have two identifications between isometries of the compactified space and symmetries on the boundary. Firstly we have the Killing spinor equation in the spacetime metric which gives a condition for the unbroken supersymmetries in the spacetime and for the case of $\mathbf{S^1} \times \mathbf{C^3}/\mathbf{Z_3}$ this gives eight unbroken supersymmetries corresponding to $N = 2$ superconformal field theories in 3 dimensions (we use the fact that compactification on $\mathbf{C^3}/\mathbf{Z_3}$ breaks 1/4 of the supersymmetry and compactification on $\mathbf{S^1}$ does not break any more supersymmetry. Secondly, the R symmetry comes from the isometry of $\mathbf{S^1}$ which gives a $U(1)$ symmetry which is just the required R symmetry for $N = 2$ in 3 dimensions.

We also need to explain the passage from the supegravity solution on $AdS_4 \times \mathbf{S^7}$ to a solution on $\mathbf{S^1} \times \mathbf{C^3}/\mathbf{Z_3}$. Although the eleven dimensional solution by promoting

[1] Generally speaking, there are many orbifolds with free or nonfree actions on $\mathbf{S}^7$ leading to different amount of supersymmetry. See recent paper by Morrison and Plesser [35]. Let us consider M2 branes at $\mathbf{C}^4/\Gamma$ singularity where the group Γ is generated by $\mathrm{diag}(\exp(2\pi i/k), \exp(-2\pi i/k), \exp(2\pi i a/k), \exp(-2\pi i a/k))$ for some relatively prime intergers a and k. If $a = 1, k = 2$ we get the maximal case $\mathcal{N} = 8$. For $a = \pm 1, k \geq 2$, one gets $\mathcal{N} = 6$ theory [36]. When Γ is a binary dihedral group, D type, singularity and we embed Γ into $SU(2) \times SU(2)$, we get $\mathcal{N} = 5$ theory [37]. When $a \neq \pm 1$, the theory has $\mathcal{N} = 4$ supersymmetry.

[2] The group actions Γ contains A type(cyclic group), D type(binary dihedral group, E type(binary tetrahedral, octahedral, icosahedral group) in general. For the AdS/CFT correspondence the global symmetry of the field theory we are considering is being given by $U(1)_R$. In the supergravity side this corresponds to an isometry of $\mathbf{S^7}$. We found that the only $\mathbf{Z_3}$ orbifold case in cyclic group is relevant to this analysis. Other orbifold cases will give rise to different global symmetry.

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D2 brane solution is not exactly M2 brane solution, in general, when the eleventh direction is compact, by taking M2 branes to be localized in the transverse directions, these two solutions are the same in M theory limit [21]. The passage from D2 to M2 is by dimensional reduction, both being extended in the (x^1, x^2) directions. By using the fact that the radius of $\mathbf{S^7}$ is proportional with N and is thus very large in the large N limit, the x^3 direction can be approximated with a circle in the large N limit.

We can now identify the gauge symmetry and the field content. The starting point is the configuration with D2 branes in $\mathbf{S^1} \times \mathbf{C^3}/\mathbf{Z_3}$, the fact that the D2 branes are at a singularity implying that the gauge group of the theory is $SU(N) \times SU(N) \times SU(N)$ and the matter content is given by fields transforming as $(\mathbf{N}, \overline{\mathbf{N}}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{N}, \overline{\mathbf{N}}) \oplus (\overline{\mathbf{N}}, \mathbf{1}, \mathbf{N})$.

We make here a connection with two related brane configurations. We firstly use the result of [32] where they discussed the $\mathcal{N} = 1$ theory in 4 dimensions. One of the main results of their work is that a configuration with D3 branes at $\mathbf{C^3}/\mathbf{Z_3}$ singularities is mapped to a Brane Box Model of intersecting NS5 and D5 branes constructed on a two-torus T^2. The map is interpreted as a T-duality along the two directions of the torus. Here we are interested in 3 dimensional $\mathcal{N} = 2$ theories so we take a Brane Box Model with intersecting NS5 and D4 branes which can be obtained from the configuration of [32] by a T-duality along a compact direction parallel with the NS5 branes which leaves unchanged the two NS5 branes.

For self-consistency, we repeat their arguments here. Let us consider the type IIA theory obtained from M theory after dimensional reduction with: NS5 branes along $(x^0, x^1, x^2, x^3, x^4, x^5)$ directions, NS'5 branes along $(x^0, x^1, x^2, x^3, x^6, x^7)$ directions and D4 branes along $(x^0, x^1, x^2, x^4, x^6)$ directions

The D4 branes are finite in the 4 and 6 directions so their low-energy effective world volume theory is $2+1$ dimensional and the supersymmetry is $1/8$ of the original supersymmetry (so we are dealing with $N = 2$ supersymmetry in 3 dimensions). We take x^4 and x^6 directions to be circles so we can make T-duality on them. We divide the x^4 and x^6 plane into a set of boxes, the number of boxes on x^4 and x^6 directions giving the gauge group and the matter content, respectively. We are interested here in the brane box of figure 7, in particular, in [32]. This is a 3×1 configuration in the x^4 and x^6 directions having the right gauge group and matter content. To study explicitly the connection between the Brane Box configuration and the one involving D2 branes at singularity we need to perform a T duality along x^4 and x^6 directions so the D4 branes are mapped into D2 branes along (x^0, x^1, x^2) on top of $\mathbf{C^3}/\mathbf{Z_3}$ as explained in page 29 of [32].

Another brane configuration equivalent to M2 branes with a transverse space $\mathbf{R^2} \times \mathbf{C^3}/\mathbf{Z_3}$ is obtained by using the duality between M theory compactified on T^2 and type IIB theory compactified on S^1. As in [38, 39, 40], M2 branes unwrapped on T^2 correspond to D3 branes wrapped on S^1 and the M5 branes wrapped on (p, q) cycles of T^2 correspond to (p, q) branes in type IIB theory and both correspond to a 2 brane in 9 dimensions. The space $\mathbf{C^3}/\mathbf{Z_3}$ can be considered as an affine cone over a projective space $\mathbf{P}^2$. By blowing up the vertex, the space can be identified with a neighborhood of $\mathbf{P}^2$ embedded in Calabi-Yau threefold, in which the normal bundle of $\mathbf{P}^2$ in the Calabi-Yau threefold is the canonical line bundle. We now have a 3-dimensional local toric geometry, where the extra circle action comes from the rotation on the phase of the normal line bundle. Now by blowing down $\mathbf{P}^2$, we obtain an affine toric variety $\mathbf{C^3}/\mathbf{Z_3}$ whose toric skeleton is given by a vertex with 3 external legs [41]. So the $(1,0),(0,1),(1,1)$ brane web configuration corresponds to $\mathbf{C^3}/\mathbf{Z_3}$ toric skeleton. If we have N D3 branes between two vertices, the gauge theory is $SU(N)$ with a chiral field transforming in the N representation.

Now, if we return to the Brane Box Model of above, we can connect it to the 3×1 model by a set of dualities we can arrive to the right web configuration which contains three web points and D3 branes between them on a circle giving thus the right gauge group and field content as explained in [38]. To see this in detail, we start in 11 dimensions from a configuration with a M2 brane in (012) directions and two Kaluza-Klein monopoles in (589,10) and (6789) directions. By reducing to 10 dimensions to obtain type IIA theory, we obtain a configuration with a D2 brane in (012) directions, a D6 brane in (0123467) directions and a KK monopole in (6789) directions. A T-duality on x^6 direction gives D3(0126), NS(012345) and D5(012347). As explained in [38], in the (x^5,x^7) plane besides the (1,0) and (0,1) lines represented by NS and D5 there is the (1,1) line so the configuration gives just a D3 brane between two webs. On the other side, if we start in 11 dimensions with a configuration with M2 (012), KK (6789) and KK (4589), by reduction to type IIA and two T-dualities on x^4,x^6 directions, one obtains a configuration with D4(1246), NS (12345) and NS (12367) which is just our Brane Box Model. In 11 dimensions the two starting brane configurations are almost identical because all the dimensions are infinite, therefore quantum results concerning Brane Box Models could be obtained by using similar results for web models [39, 40].

3.2. Surviving Kaluza-Klein modes

We now proceed to identify the Kaluza-Klein modes that survive after the orbifolding procedure. We also determine the operators in the conformal field theory side to which the Kaluza-Klein modes couple. These operators are built as combinations of the scalars which enter the theory. These are of three types: the scalar part of the chiral multiplet coming from the 4 dimensional chiral multiplet, the real scalar corresponding to the component of the $D=4$ vector potential in the reduced direction and the third one is obtained in the bulk of the Coulomb branch by dualising the gauge fields. The last two are combined into a chiral superfield $\Phi^j, j=1,2,3$. We denote the matter multiplets by U,V,W. We assign the $U(1)_R$ charges 1/2 to Φ^j and 2/3 to U,V,W. By considering the relation between the dimension of chiral operators and $U(1)_R$ symmetry charges as [42, 43, 44]:

$$\Delta = |R| \qquad \text{or} \qquad |R|+1\,, \tag{3.1}$$

we take the dimension of Φ^j as $1/2$ and the one of U, V, W as $2/3$. Now we proceed to identify the surviving Kaluza-Klein modes.

• The $k = 2$ Kaluza-Klein particle in (2.1) transforms in the $\mathbf{35_v}$ of $SO(8)$. By decomposing $\mathbf{35_v}$ into the representation of $SU(3)_\Gamma \times U(1)_R$, we obtain $\mathbf{1_0}$ which is invariant under Γ with right $U(1)_R$ quantum number. We list $SO(8) \to SU(3)_\Gamma \times U(1)_R$ branching rules in the Appendix with the help of [45]. We expect that a dimension 1 chiral primary operator, according to (2.2), to live in the boundary SCFT. This Kaluza-Klein mode couples to the dimension 1 chiral operator $\Sigma_{i=1}^3 \mathrm{Tr}(\Phi^i \widetilde{\Phi}_i)$ where the index i enumerates the three gauge groups $SU(N) \times SU(N) \times SU(N)$. Here $\widetilde{\Phi}_i$ is the field conjugate to Φ and has $U(1)_R$ charge $-1/2$.

• The The $k = 3$ Kaluza-Klein particle in (2.1) transforms in the $\mathbf{112_v}$ of $SO(8)$. There is no invariant state under the Γ projection with right $U(1)_R$ charge.

• The $k = 4$ Kaluza-Klein particle in (2.1) transforms in the $\mathbf{294_v}$ of $SO(8)$. The $\mathbf{1_2}, \mathbf{1_{-2}}, \mathbf{8_2}, \mathbf{8_{-2}}, \mathbf{10_{-2}}$ and $\overline{\mathbf{10}}_\mathbf{2}$ are invariant under Γ so these states will survive the projection. From their quantum numbers, this Kaluza-Klein modes couple respectively to $\mathrm{Tr}(U^i V^j W^k)$ and $\mathrm{Tr}(\widetilde{U}^i \widetilde{V}^j \widetilde{W}^k)$ where here we take all the possible combinations of i, j and k by using the product rules $\mathbf{3} \times \mathbf{3} = \overline{\mathbf{3}} \oplus \mathbf{6}, \mathbf{3} \times \overline{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}, \mathbf{3} \times \mathbf{6} = \mathbf{8} \oplus \mathbf{10}$. So all the surviving Kaluza-Klein modes can couple to chiral primary operators in the SCFT.

• The $k = 5$ Kaluza-Klein particle in (2.1) transforms in the $\mathbf{672'_v}$ of $SO(8)$. There is no invariant state under the Γ projection.

• The $k = 6$ Kaluza-Klein particle in (2.1) transforms in the $\mathbf{1386_v}$ of $SO(8)$. The $\mathbf{1_2}, \mathbf{1_{-2}}, \mathbf{8_2}, \mathbf{8_{-2}}, \mathbf{10_{-2}}, \mathbf{10_2}, \overline{\mathbf{10}}_\mathbf{2}, \overline{\mathbf{10}}_\mathbf{-2}, \mathbf{27_2}, \mathbf{27_{-2}}, \mathbf{28_{-2}}, \overline{\mathbf{28}}_\mathbf{2}, \mathbf{35_{-2}}$ and $\overline{\mathbf{35}}_\mathbf{2}$ are invariant under Γ so these states will survive the projection. They have right $U(1)_R$ charge. These Kaluza-Klein modes can couple to dimension 3 operators which are similar to the ones obtained at $k = 4$, being as $\mathrm{Tr}(U^i V^j W^k \Phi^i \widetilde{\Phi}_i)$. The last six Kaluza-Klein modes cannot be written in terms of the short distance fields. The only gauge invariant possibility would involve $\mathrm{Tr}(U^i V^j W^k)^2$ which is of dimensions 4 so being greater than 3.

If we look now to the pseudoscalar tower, we see that for $k = 1$ ($\Delta = 2$) we have $\mathbf{1_2}, \mathbf{1_{-2}}$ as surviving states and for $k = 3$ we have the $\mathbf{1_2}, \mathbf{1_{-2}}, \mathbf{8_2}, \mathbf{8_{-2}}$ as surviving states, all of them can be written as gauge invariant combinations of operators in SCFT. The massless vector and the graviton couple to the global $U(1)_R$ symmetry current and to the energy momentum respectively, their dimensions being protected from quantum corrections.

4. Conclusion

In this paper we have obtained a part of the spectrum of chiral primary operators in the superconformal multiplets of $\mathcal{N} = 2, D = 3$ SCFT by using the field theory on the M2 worldvolume. We performed only calculations in supergravity so we have just obtained the untwisted sector results. In order to smoothen out the singularity of the

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spacetime, we need to consider the full M theory. It would be interesting to study the possible twisted modes in the full M theory.

We have used the map between M2 branes at $\mathbf{C^3}/\mathbf{Z_3}$ and Brane Box Models. Another configuration giving M2 branes at $\mathbf{C^3}/\mathbf{Z_3}$ is the one with D3 branes between three webs of (0,1), (1, 0) and (1, 1) branes which are on a circle. We have identified a connection between a Brane Box Model and a web model which could help to enlarge the quantum description of both.

Our approach to obtain the spectrum of chiral primary operators is similar to the one used in [9] for D3 branes at $\mathbf{C^3}/\mathbf{Z_3}$ which gives $\mathcal{N} = 1$ in $D = 4$. It would be interesting to obtain $\mathcal{N} = 2$ in $D = 3$ by considering M theory compactified on a Calabi-Yau 4-fold. One discussion is made in [41] where a Calabi-Yau 4-fold is described in terms of a tetrahedron corresponding to certain (p, q, r) 4 - branes where the (p, q, r) vector label the Kaluza-Klein monopoles obtained when M theory is compactified on T^3.

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A. $SO(8)$ branching rule

Fields	$SO(8)$ Dynkin label	$SU(3)_\Gamma \times U(1)_R$
vector	$(0,1,0,0)$: $\mathbf{28}$	$\mathbf{1}_{0,0} \oplus \mathbf{3}_{2/3,2/3,-4/3} \oplus \overline{\mathbf{3}}_{-2/3,-2/3,4/3} \oplus \mathbf{8}_0$
scalar	$(2,0,0,0)$: $\mathbf{35_v}$	$\mathbf{1}_{0,-2,2} \oplus \mathbf{3}_{2/3,-4/3} \oplus \overline{\mathbf{3}}_{-2/3,4/3}$ $\oplus \mathbf{6}_{-2/3} \oplus \overline{\mathbf{6}}_{2/3} \oplus \mathbf{8}_0$
pseudoscalar	$(0,0,2,0)$: $\mathbf{35_c}$	$\mathbf{1}_{0,-2,2} \oplus \mathbf{3}_{2/3,-4/3} \oplus \overline{\mathbf{3}}_{-2/3,4/3}$ $\oplus \mathbf{6}_{-2/3} \oplus \overline{\mathbf{6}}_{2/3} \oplus \mathbf{8}_0$
scalar	$(3,0,0,0)$: $\mathbf{112_v}$	$\mathbf{1}_{-1,1,-3,3} \oplus \mathbf{3}_{-1/3,5/3,-7/3} \oplus \overline{\mathbf{3}}_{1/3,-5/3,7/3}$ $\oplus \mathbf{6}_{1/3,-5/3} \oplus \overline{\mathbf{6}}_{-1/3,5/3} \oplus \mathbf{8}_{-1,1} \oplus \mathbf{10}_{-1}$ $\oplus \overline{\mathbf{10}}_1 \oplus \mathbf{15}_{-1/3} \oplus \overline{\mathbf{15}}_{1/3}$
pseudoscalar	$(1,0,2,0)$: $\mathbf{224_{cv}}$	$\mathbf{1}_{-1,-1,1,1,-3,3}$ $\oplus \mathbf{3}_{-1/3,-1/3,-1/3,5/3,5/3,-7/3,-7/3}$ $\oplus \overline{\mathbf{3}}_{1/3,1/3,1/3,-5/3,-5/3,7/3,7/3}$ $\oplus \mathbf{6}_{1/3,1/3,-5/3,-5/3} \oplus \overline{\mathbf{6}}_{-1/3,-1/3,5/3,5/3}$ $\oplus \mathbf{8}_{-1,-1,-1,1,1,1} \oplus \mathbf{10}_{-1} \oplus \overline{\mathbf{10}}_1$ $\oplus \mathbf{15}_{-1/3,-1/3} \oplus \overline{\mathbf{15}}_{1/3,1/3}$
scalar	$(4,0,0,0)$: $\mathbf{294_v}$	$\mathbf{1}_{0,-2,2,-4,4} \oplus \mathbf{3}_{2/3,-4/3,8/3,-10/3}$ $\oplus \overline{\mathbf{3}}_{-2/3,4/3,-8/3,10/3} \oplus \mathbf{6}_{-2/3,4/3,-8/3}$ $\oplus \overline{\mathbf{6}}_{2/3,-4/3,8/3} \oplus \mathbf{8}_{0,-2,2} \oplus \mathbf{10}_{0,-2} \oplus \overline{\mathbf{10}}_{0,2}$ $\oplus \mathbf{15}_{2/3,-4/3} \oplus \mathbf{15}'_{-4/3} \oplus \overline{\mathbf{15}'}_{4/3}$ $\oplus \overline{\mathbf{15}}_{-2/3,4/3} \oplus \mathbf{24}_{2/3} \oplus \overline{\mathbf{24}}_{-2/3} \oplus \mathbf{27}_0$
scalar	$(5,0,0,0)$: $\mathbf{672'_v}$	$\mathbf{1}_{-1,1,-3,3,-5,5} \oplus \mathbf{3}_{-1/3,5/3,-7/3,11/3,-13/3}$ $\oplus \overline{\mathbf{3}}_{1/3,-5/3,7/3,-11/3,13/3} \oplus \mathbf{6}_{1/3,-5/3,7/3,-11/3}$ $\oplus \overline{\mathbf{6}}_{-1/3,5/3,-7/3,11/3} \oplus \mathbf{8}_{-1,1,-3,3} \oplus \mathbf{10}_{-1,1,-3}$ $\oplus \overline{\mathbf{10}}_{-1,1,3} \oplus \mathbf{15}_{-1/3,5/3,-7/3} \oplus \overline{\mathbf{15}}_{1/3,-5/3,7/3}$ $\oplus \mathbf{15}'_{-1/3,-7/3} \oplus \overline{\mathbf{15}'}_{1/3,7/3}$ $\oplus \overline{\mathbf{21}}_{-5/3} \oplus \mathbf{21}_{5/3} \oplus \mathbf{24}_{-1/3,5/3} \oplus \overline{\mathbf{24}}_{1/3,-5/3}$ $\oplus \mathbf{35}_{-1} \oplus \overline{\mathbf{35}}_1 \oplus \mathbf{27}_{-1,1} \oplus \overline{\mathbf{42}}_{1/3} \oplus \mathbf{42}_{-1/3}$

Table 1: The superconformal multiplets, their $SO(8)$ Dynkin labels and branching rules for $SU(3)_\Gamma \times U(1)_R$. $U(1)_R$ charges are given in the subscript of $SU(3)_\Gamma$ representations. For simplicity, we denote $\mathbf{1_0} \oplus \mathbf{1_0}$ by $\mathbf{1}_{0,0}$ and so on.

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Fields	$SO(8)$ Dynkin label	$SU(3)_\Gamma \times U(1)_R$
pseudoscalar	$(2,0,2,0):\quad \mathbf{840'_s}$	$\mathbf{1}_{0,0,0,-2,-2,2,2,-4,4}$ $\oplus\mathbf{3}_{2/3,2/3,2/3,2/3,-4/3,-4/3,-4/3,-4/3}$ $\oplus\mathbf{3}_{8/3,8/3,-10/3,-10/3}$ $\oplus\overline{\mathbf{3}}_{-2/3,-2/3,-2/3,-2/3,4/3,4/3,4/3,4/3}$ $\oplus\overline{\mathbf{3}}_{-8/3,-8/3,10/3,10/3}$ $\oplus\mathbf{6}_{-2/3,-2/3,-2/3,-2/3,4/3,4/3,4/3,-8/3,-8/3,-8/3}$ $\oplus\overline{\mathbf{6}}_{2/3,2/3,2/3,2/3,-4/3,-4/3,-4/3,8/3,8/3,8/3}$ $\oplus\mathbf{8}_{0,0,0,0,0,0,-2,-2,-2,-2,2,2,2,2}$ $\oplus\mathbf{10}_{0,0,-2,-2}\oplus\overline{\mathbf{10}}_{0,0,2,2}$ $\oplus\mathbf{15}_{2/3,2/3,2/3,2/3,-4/3,-4/3,-4/3,-4/3}$ $\oplus\overline{\mathbf{15}}_{-2/3,-2/3,-2/3,-2/3,4/3,4/3,4/3,4/3}$ $\oplus\mathbf{15'}_{-4/3}\oplus\overline{\mathbf{15'}}_{4/3}\oplus\overline{\mathbf{24}}_{-2/3,-2/3}\oplus\mathbf{24}_{2/3,2/3}$ $\oplus\mathbf{27}_{0,0,0}$
scalar	$(6,0,0,0):\quad \mathbf{1386_v}$	$\mathbf{1}_{0,-2,2,-4,4,-6,6}$ $\oplus\mathbf{3}_{2/3,-4/3,8/3,-10/3,14/3,-16/3}$ $\oplus\overline{\mathbf{3}}_{-2/3,4/3,-8/3,10/3,-14/3,16/3}$ $\oplus\mathbf{6}_{-2/3,4/3,-8/3,10/3,-14/3}$ $\oplus\overline{\mathbf{6}}_{2/3,-4/3,8/3,-10/3,14/3}$ $\oplus\mathbf{8}_{0,-2,2,-4,4}\oplus\mathbf{10}_{0,-2,2,-4}\oplus\overline{\mathbf{10}}_{0,-2,2,4}$ $\oplus\mathbf{15}_{2/3,-4/3,8/3,-10/3}\oplus\mathbf{15'}_{2/3,-4/3,-10/3}$ $\oplus\overline{\mathbf{15}}_{-2/3,4/3,-8/3,10/3}\oplus\overline{\mathbf{15'}}_{-2/3,4/3,10/3}$ $\oplus\mathbf{21}_{2/3,8/3}\oplus\overline{\mathbf{21}}_{-2/3,-8/3}\oplus\mathbf{24}_{2/3,-4/3,8/3}$ $\oplus\overline{\mathbf{24}}_{-2/3,4/3,-8/3}\oplus\mathbf{27}_{0,-2,2}\oplus\mathbf{28}_{-2}\oplus\overline{\mathbf{28}}_{2}$ $\oplus\mathbf{35}_{0,-2}\oplus\overline{\mathbf{35}}_{0,2}\oplus\mathbf{42}_{2/3,-4/3}\oplus\overline{\mathbf{42}}_{-2/3,4/3}$ $\oplus\mathbf{48}_{-4/3}\oplus\overline{\mathbf{48}}_{4/3}\oplus\mathbf{60}_{2/3}\oplus\overline{\mathbf{60}}_{-2/3}\oplus\mathbf{64}_{0}$

Table 1: (Continued)

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